Perceptions of Mathematics from within and without

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Abstract
This is a discussion article concerning the way we as mathematicians see our subject from within, and the way in which Joe(anna) public sees this subject, from without. The perceived difficulty and decline in the popularity of mathematics is affecting courses in both Schools and Universities. The claim of this article is that it is not its nature or image that makes it so but rather the "language" and "relevance" of maths. A more descriptive approach in books and lecturing style would benefit both the mathematical community and students. An approach which motivates and contextualises the relevance of the mathematics being taught will assist in its absorption. The excitement we feel in research must be evident in mathematical literature. in a way that opens the beauty of this subject to the reader.

1. Introduction

How is mathematics perceived by society and how are we as mathematicians viewed? How do we portray our subject, and how do we perceive its difficulties? Why does the average intelligent person think that mathematics is a dull, difficult, unintelligible subject, outside of their grasp; indeed that its theories are out-dated and are of little use or importance to them or their lives. These are some of the questions which we must ask; this article makes no claim to a solution to these questions but it is hoped it may encourage discussion amongst the mathematical community. Of course, it could be argued that mathematics today is such a broad subject, that we should talk about distinct disciplines within this main title. This is not the within the remit of this article; from without mathematics is viewed as one entity. The main concerns are as the title of this article suggests.

1.1 From without

If you were to ask a member of the public to paint an image of a mathematician, we are all to aware of the image that society in general is likely to conjure up : that of an elderly man, bespectacled and undoubtedly appearing absent-minded (no disrespect to the mathematicians that indeed do conform to this image). (See for example [2]). It could however be argued that if one's occupation can be guessed, one's camouflage really needs improving! However we are proposing for a moment that we all don strange outfits; what is suggested is that this image is not allowed to be the image of our subject.

Suppose a second question is posed to your Joe(anna) public : "what is your perception of mathematics?" I think that none of us would be surprised to hear cries of "Numbers, Algebra, Arithmetic, x's and y's ." (See [3]). We would be woken from our *ennui ´* by comments such as "If you add or subtract two integers, you get an integer solution " (usual addition here). Why is it that the general public doesn't look at the properties we study, worse still, is unaware that in fact our subject is involved with concepts, properties, realities that become abstractions. The mathematical community must accept some of the responsibility for this rather limited view of the subject. From a very early age Joe(anna) public is taught not how we as mathematicians actually think; we train people in the way we think they will understand. We assume that what we do is some special gift, requiring a huge mental exertion, and we really cannot begin to expect everyone else to be as gifted. We teach how we perform operations rather than explaining how more generally we might look at the same problem. We often teach repetition and not understanding simply because it is much easier and less preparation is needed. To begin to teach in a way that tries to motivate the subject matter by showing our intuition, explaining how we could see the problem - to almost show its bare bones, which exactly what we need to do to solve it might help in the training of future generations. This would require changes, as we shall suggest below. Skemp has argued that students and teachers soemtimes mismatch their communications, resulting in misunderstandings. A student believes he has understood a topic, and s/he may have done, but the point of understanding of language is about different things and on a different level.

Skemp says :

> *A teacher may say "You may think*
> *you understand, but you don't really," he (the student) would not agree. "Of course*
> *I do. Look; I've got all these answers right."*

See [4, page 21].

1.2 From within

To a practitioner, mathematics is very much alive; it delights when one least expects it; it does take time to establish insight with it, but that's true of many things. As mathematicians we feel our subject requires a mental energy; the sheer pleasure of performing at speed and the inter-connections we use quite naturally without any conscious effort is part of the reason we enjoy our subject. But our subject should become less of an exclusive domain. It is imperative that we reconsider the path our subject develops along. The beauty, elegance and delight we see and feel about mathematics is not something we should be afraid to share with society. I refer you to G. H. Hardy's classic for a very elegant discussion of this topic and more [2].

Are we mathematicians at fault? Is it that we feel that mathematics is a sacred domain which needs to remain remote in order that it retains its awe and respect? More importantly that we maintain our high academic esteem. It could be argued that in fact we are on the wrong path; our subject is proving less understood and less popular and in real danger of being radically cut back both at 16+ and undergraduate level. It is clearly in our interests to make mathematics more accessible. It is also in the interests prospective undergraduates to capture and inspire young people to pursue a mathematically-biased subject. This will benefit the mathematical community, much more so than society at large.

There have been many recent attempts to make science and mathematics more popular, not least including the excellent popular mathematics lectures run by the London Mathematical Society, (see, for example, http://www.lms.ac.uk/content/previous-popular-lectures), but more could be done by the mathematical community. To suggest that the very reason we enjoy maths and get a "buzz" from exploring it is the very thing we should be promoting and trying to open to society in general might be considered extreme but it might be a possible avenue to explore. It would be wrong to claim that the decline of interest in maths is simply due to the nature and image of the subject or our approach to teaching; current values in society must play a role and in particular students often have a clear vocation in mind or at least a direction and the idea of studying a subject which may be useful in getting a job is not nearly as good as a subject that will get a job.

Skemps' arguments are very interesting on the "psychology" of teaching and in particular, learning Mathematics. He states that he began researching this topic after some years of teaching mathematics and feeling that some students might have achieved more, but soemthing was missing. In [5] he discusses how he went about finding a learning method, that might remedy this situation. (We won't go into the details in this short paper, but leave you to read his text).

2. Towards a change using Language

How can we redress the balance, and make the subject both more accessible and better understood? Some mathematicians are moving away from teaching lemma, theorem, corollary as students are not prepared for this and a move towards rigour-within-an-example is the main framework of delivery. Perhaps this is successful, but it surely side-steps the real problem. It feels wrong to teach a set of examples and from this to define the more general state of play and surely this would not stimulate the enquiring mathematician.

A slight change in presentation is all that is suggested; the impact of this alone will make a considerable difference to correct this balance. It is the very "language" of mathematics which makes it so unapproachable to the general public and in particular, to students. If a slight change of emphasis was introduced to our style of teaching at all levels, including undergraduate work, a far greater depth of understanding could be achieved, and syllabuses could still be covered. You may well ask "how exactly is this slight change of focus achieved?" It is not suggested that we start cracking jokes in the middle of a proof. (In a lecture we can of course do this but in a book somehow it just looks crass). The proposition is two-fold : nothing more than this : when you are teaching a formal proof or definition, give the very essence of its meaning; write the ideas that are contained therein as part of the explanation you give. That isn't to say that we should use no symbolic language; simply that more description should be used. To avoid using formal language entirely would be self-defeating, but let the descriptive approach be first adopted to get the idea across. Once the class has grasped the core concept then this is the point to begin to formalise the idea. By using a descriptive approach we can motivate the subject to a wide range of ability and still leave room for the above average to be challenged.

Surely we want students to understand the ideas underlying the definition or technique? If the teaching of mathematics has a main emphasis on developing concepts, the subject can easily be viewed in a much more "student-friendly" way. The way mathematicians write formally sound arguments is merely the LANGUAGE of mathematics. The language must be the vehicle and not the reason for the subject.

You may wonder whether this change of emphasis can really be so fundamentally transforming. Well, it certainly is not new. One only has to consider the works of mathematicians at the turn of the century. Mathematical language was far less rigorous and established than it is today. For example the works of Borel, Lusin or Baire, Lebesgue and many others. See for example, [6] Their works are to some extent essays. It is this that makes them accessible and delightful to read. I hasten to add that Formal Language systems do act as tools for progress and without such research mathematics would indeed be impeded. However, we have travelled so far in new fields of development, that the language itself is perceived from without as being the driving force. Such a comment is nonsense to a mathematician, but this is indeed the image that prevails. It is wrong, and it must be changed if our subject is to become more popular, more widely enjoyed and continue to break new ground.

If text books were written in a more descriptive style, students would find them far more accessible and would understand why we enjoy the subject. It would appeal to a far wider audience and be something Joe(anna) public could relate to. It does not follow that there will be less mathematics, but simply the material would be presented in a way that "talks" about the topics under discussion and why a proof takes certain steps. A Technical Author colleague has coined the phrase "start at the user-end". Who knows, students may even be more motivated!

Below we give some examples of how mathematics can be put into an every day context and introduced without heavy rigour. There are many other examples of making Mathematics real and living. It would not be appropriate to give an example of a proof here in a more descriptive way as most school's would not teach such formal results but the author does have examples if any reader wants to see some, send an e-mail.

2.1 Examples - Language driven- contextualised

Examples from the class room

Using descrptive langauge for abstract concepts we believe will help enormously with developing a wider appreciation of the subject. For example, suppose you were introducing metric spaces. Introduce the axioms using a specific example. Such as : suppose we measure the distance -as the crow flies- between two towns on a map (Pythagorean distance). The axioms follow naturally : the distance from a town to itself is zero, and the distance between two distinct towns is strictly greater than zero. Obviously so long as both points are fixed, the distance between two places is also finite. The other axioms follow accordingly. Then state the usual notation. It is clear that you could measure distance in ways other than using the Pythagorean notion.

Introducing Groups in such a way is well known. Why not introduce Rings, Vector spaces etc, etc, like this too, using everyday situations? Any suggestions?

2.3 Introducing Non- Euclidean geometry – The hair dresser and dress-makers' problem

Suppose you were cutting your daughter's long hair into a shorter bob style where the hair is the same length all the way around. How exactly would you do this? One way would be to pull all the hair to say the back and so arrange the hair into the shape of a level plane and cut the bottom of this plane in a straight line perpendicular with the horizontal. Would the hair be level all around? Well you could try this but if you asked a hair dresser they would without hesitation tell you would never make a hairdresser! Think of the head as a globe and on this try to imagine some level lines of latitude - a hair dresser would fix a length from near the north pole of the skull (eg the crown) and use this as a measure. As the hair is cut along any line of latitude then it will be level all the way around - but if you then pull a section into a plane you will notice that the length does not appear to be level on the plane. A mathematician would say that this is a distinguishing feature between Euclidean and Non-Euclidean geometry. The hair dresser may not know the mathematical explanation but they certainly would know the mathematical principles involved.

A similar example follows for dress-makers. Designing an object of clothing requires a 2-dimensional piece arrangement of a pattern, all pieces lying in a 2-dimensional plane. The pattern is placed on a 2-dimensional plane of fabric to be cut and sewed together. The resulting object, such as a dress, is now a curved 2-dimensional surface, which has quite different geometry and curvature to the original plane-object. If we were to think of measuring distances in the the original planar object (the pattern and fabric) this would be Euclidean distances, but in the final dress, we would need a different method of measuring distances, taking into account the now-curved (non-Euclidean) fabric.

2.4 Introducing polar-co-ordinates and measuring distance : The Engineer's dilemma a lathe cuts circular paths.

Consider a lathe which cuts metal or cuts indentations into metal. It has two possible movements - it can move in and out in a straight line or it can rotate in a circular orbit. These two main features make it an ideal example for introducing the co-ordinate system of polar co-ordinates - the radial movement in and out is the r value and the angle it rotates is the value of q and so any position of the lathe cutting edge is defined in terms of (r, q). See for example, [1, Chapter 7]. Now suppose the lathe is connected to a computer and you have used standard macros to enable the lathe to engrave on a flat sheet of metal. When you take the sheet off the workbench you see that the writing is in arcs such as :

This is engraved using a circular lathe

2.5 Matrices and a Computer Screen

For example, suppose we were introducing matrices for the first time. We could naturally say they are a generalisation of a single number, and that would be of interest and motivate mathematicians, but would it make any impression on an average student? Linking the ideas to computer games would make it more lively and "put it into context' as my educational colleagues would say. The position of each point on the screen (pixel) would be a vector relative to say one corner of the screen. Each movement of an a point or object across the screen would be a matrix transformation, be it a rotation or enlargement etc. Suddenly matrices would seem important to the student's everyday awareness. Obviously for the more advanced class perspective could be introduced.

Conclusion

Many peoples earliest experiences of Mathematics are short of being extremely negative, demoralising and humiliating. It is somewhat heartbreaking to think that this is how Joe(anna) public sees Mathematics at an early age; this usually continues with them throughout their lives. Our experiences, as Mathematicians, are so very different. For things to change, then we need to look at primary education and secondary education; we must exert influence in these spheres. Nonetheless, we produce the graduates of today who will be the Maths teachers of tomorrow. The change must start and come from us initially. This has to be a top-down approach.

References

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See also See http://www.skemp.org.uk/

[6] Emile Borel, 1871 – 1956. http://en.wikipedia.org/wiki/%C3%89mile_Borel

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